The Plane!
Modeling Linear Situations

Vocabulary
Define each term in your own words.

1. first differences
   First differences are the differences between successive data points.

2. solution
   The solution of a linear equation is any value that makes an open sentence true.

3. intersection point
   The intersection point is the point on the graph where one line crosses another.

Problem Set
Identify the independent and dependent quantities in each problem situation. Then write a function to represent the problem situation.

1. Nathan is riding his scooter to school at a rate of 6 miles per hour.
   The distance Nathan travels depends on the time. Distance, \( D \), is the dependent quantity and time, \( t \), is the independent quantity.
   \[ D(t) = 6t \]

2. Sophia is walking to the mall at a rate of 3 miles per hour.
   The distance Sophia travels depends on the time. Distance, \( D \), is the dependent quantity and time, \( t \), is the independent quantity.
   \[ D(t) = 3t \]

3. Mario is stuffing envelopes with invitations to the school’s Spring Carnival. He stuffs 5 envelopes each minute.
   The total number of envelopes Mario stuffs depends on the time. The total number of envelopes, \( E \), is the dependent quantity and time, \( t \), is the independent quantity.
   \[ E(t) = 5t \]
4. Shanise plays on the varsity soccer team. She averages 4 goals per game.
   The total number of goals Shanise scores depends on the number of games she plays. The total number of goals scored, \( S \), is the dependent quantity and the number of games played, \( g \), is the independent quantity.
   \[ S(g) = 4g \]

5. The football booster club sells hot chocolate during the varsity football games. Each cup of hot chocolate costs $2.
   The amount of money the booster club earns depends on the number of cups sold. The amount of money, \( M \), is the dependent quantity and the number of cups sold, \( c \), is the independent quantity.
   \[ M(c) = 2c \]

6. The basketball booster club sells t-shirts at the varsity basketball games. Each t-shirt costs $12.
   The amount of money the booster club earns depends on the number of t-shirts sold. The amount of money, \( M \), is the dependent quantity and the number of t-shirts sold, \( t \), is the independent quantity.
   \[ M(t) = 12t \]

Use each scenario to complete the table of values and calculate the unit rate of change.

7. Miguel is riding his bike to lacrosse practice at a rate of 7 miles per hour.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Distance</td>
</tr>
<tr>
<td>Units</td>
<td>miles</td>
</tr>
<tr>
<td>Expression</td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>Distance</td>
</tr>
<tr>
<td>hours</td>
<td>miles</td>
</tr>
<tr>
<td>( t )</td>
<td>( 7t )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>3.5</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>1.5</td>
<td>10.5</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
</tbody>
</table>

\( (0.5, 3.5) \) and \( (1, 7) \)
\[
\frac{7 - 3.5}{1 - 0.5} = \frac{3.5}{0.5} = \frac{7}{1}
\]
The unit rate of change is 7.
8. Jada is walking to school at a rate of 2 miles per hour.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td>Units</td>
<td>hours</td>
</tr>
<tr>
<td>Expression</td>
<td>( t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1.25</td>
<td>2.5</td>
</tr>
<tr>
<td>1.5</td>
<td>3</td>
</tr>
</tbody>
</table>

\[
\frac{0.5 - 0.25}{0.5 - 0.25} = \frac{0.5}{0.25} = \frac{2}{1}
\]

The unit rate of change is 2.

9. Noah is stuffing envelopes with invitations to the school's Harvest Festival. He stuffs 4 envelopes each minute.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td>Units</td>
<td>minutes</td>
</tr>
<tr>
<td>Expression</td>
<td>( t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Number of Envelopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>25</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
\frac{40 - 20}{10 - 5} = \frac{20}{5} = \frac{4}{1}
\]

The unit rate of change is 4.
10. Terell plays on the varsity basketball team. He averages 12 points per game.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Units</td>
</tr>
<tr>
<td>Number of games played</td>
<td>games</td>
</tr>
<tr>
<td>Total number of points scored</td>
<td>points</td>
</tr>
<tr>
<td>Expression</td>
<td>Expression</td>
</tr>
<tr>
<td>$g$</td>
<td>$12g$</td>
</tr>
</tbody>
</table>

The unit rate of change is 12.

11. The volleyball boosters sell bags of popcorn during the varsity matches to raise money for new uniforms. Each bag of popcorn costs $3.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Units</td>
</tr>
<tr>
<td>Number of bags of popcorn sold</td>
<td>bags</td>
</tr>
<tr>
<td>Amount of money raised</td>
<td>dollars</td>
</tr>
<tr>
<td>Expression</td>
<td>Expression</td>
</tr>
<tr>
<td>$b$</td>
<td>$3b$</td>
</tr>
</tbody>
</table>

The unit rate of change is 3.
12. The football boosters sell hooded sweatshirts to raise money for new equipment. Each sweatshirt costs $18.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td></td>
</tr>
<tr>
<td>Units</td>
<td></td>
</tr>
<tr>
<td>Expression</td>
<td></td>
</tr>
<tr>
<td>Number of sweatshirts sold</td>
<td>Amount of money raised</td>
</tr>
<tr>
<td>sweatshirts</td>
<td>dollars</td>
</tr>
<tr>
<td>s</td>
<td>18s</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>180</td>
</tr>
<tr>
<td>20</td>
<td>360</td>
</tr>
<tr>
<td>30</td>
<td>540</td>
</tr>
<tr>
<td>40</td>
<td>720</td>
</tr>
</tbody>
</table>

Identify the input value, the output value, and the rate of change for each function.

13. Belinda is making greeting cards. She makes 4 cards per hour. The function $C(t) = 4t$ represents the total number of cards Belinda makes as a function of time.

   The input value is $t$.
   The output value is $4t$.
   The rate of change is 4.

14. Owen is riding his bike to his friend's house at a rate of 6 miles per hour. The function $D(t) = 6t$ represents the distance Owen rides as a function of time.

   The input value is $t$.
   The output value is $6t$.
   The rate of change is 6.
15. Rochelle is shopping for earrings. Each pair of earrings costs $15 dollars. The function \( C(e) = 15e \) represents the total cost of the earrings as a function of the number of pairs of earrings Rochelle buys.
   The input value is \( e \).
   The output value is \( 15e \).
   The rate of change is 15.

16. Lavon is driving to visit a college campus. He is traveling 65 miles per hour. The function \( D(t) = 65t \) represents the total distance he travels as a function of time.
   The input value is \( t \).
   The output value is \( 65t \).
   The rate of change is 65.

17. Kiana is selling coupon books to raise money for her school. Each coupon book cost $35. The function \( M(b) = 35b \) represents the total amount of money raised as a function of the number of coupon books sold.
   The input value is \( b \).
   The output value is \( 35b \).
   The rate of change is 35.

18. Cisco mows lawns in his neighborhood to earn money. He earns $16 for each lawn. The function \( A(m) = 16m \) represents the total amount of money earned as a function of the number of lawns mowed.
   The input value is \( m \).
   The output value is \( 16m \).
   The rate of change is 16.
Solve each function for the given input value. The function \( A(t) = 7t \) represents the total amount of money in dollars Carmen earns babysitting as a function of time in hours.

19. \( A(3) = \) 
\[
A(3) = 7(3) \\
= 21
\]
Carmen earns $21 when she babysits for 3 hours.

20. \( A(2) = \) 
\[
A(2) = 7(2) \\
= 14
\]
Carmen earns $14 when she babysits for 2 hours.

21. \( A(5) = \) 
\[
A(5) = 7(5) \\
= 35
\]
Carmen earns $35 when she babysits for 5 hours.

22. \( A(4.5) = \) 
\[
A(4.5) = 7(4.5) \\
= 31.5
\]
Carmen earns $31.50 when she babysits for 4.5 hours.

23. \( A(3.5) = \) 
\[
A(3.5) = 7(3.5) \\
= 24.5
\]
Carmen earns $24.50 when she babysits for 3.5 hours.

24. \( A(6) = \) 
\[
A(6) = 7(6) \\
= 42
\]
Carmen earns $42 when she babysits for 6 hours.
Use the graph to determine the input value for each given output value. The function \( D(t) = 40t \) represents the total distance traveled in miles as a function of time in hours.

- **25.** \( D(t) = 120 \)  
  \( t = 3 \)

- **26.** \( D(t) = 320 \)  
  \( t = 8 \)

- **27.** \( D(t) = 240 \)  
  \( t = 6 \)

- **28.** \( D(t) = 160 \)  
  \( t = 4 \)

- **29.** \( D(t) = 80 \)  
  \( t = 2 \)

- **30.** \( D(t) = 400 \)  
  \( t = 10 \)
What Goes Up Must Come Down
Analyzing Linear Functions

Problem Set

Complete the table to represent each problem situation.

1. A hot air balloon cruising at 1000 feet begins to ascend. It ascends at a rate of 200 feet per minute.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td></td>
<td>Height</td>
</tr>
<tr>
<td>Units</td>
<td>minutes</td>
</tr>
<tr>
<td></td>
<td>feet</td>
</tr>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>1400</td>
</tr>
<tr>
<td>4</td>
<td>1800</td>
</tr>
<tr>
<td>6</td>
<td>2200</td>
</tr>
<tr>
<td>8</td>
<td>2600</td>
</tr>
<tr>
<td>Expression</td>
<td>t</td>
</tr>
<tr>
<td></td>
<td>$200t + 1000$</td>
</tr>
</tbody>
</table>
2. A bathtub contains 10 gallons of water. The faucet is turned on and fills the tub at a rate of 5.25 gallons per minute.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td>Units</td>
<td>Volume</td>
</tr>
<tr>
<td></td>
<td>minutes</td>
</tr>
<tr>
<td></td>
<td>gallons</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>15.25</td>
</tr>
<tr>
<td>3</td>
<td>25.75</td>
</tr>
<tr>
<td>5</td>
<td>36.25</td>
</tr>
<tr>
<td>7</td>
<td>46.75</td>
</tr>
<tr>
<td>Expression</td>
<td>$t$</td>
</tr>
<tr>
<td></td>
<td>$5.25t + 10$</td>
</tr>
</tbody>
</table>

3. A helicopter flying at 4125 feet begins its descent. It descends at a rate of 550 feet per minute.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td>Units</td>
<td>Height</td>
</tr>
<tr>
<td></td>
<td>minutes</td>
</tr>
<tr>
<td></td>
<td>feet</td>
</tr>
<tr>
<td>0</td>
<td>4125</td>
</tr>
<tr>
<td>1</td>
<td>3575</td>
</tr>
<tr>
<td>2</td>
<td>3025</td>
</tr>
<tr>
<td>3</td>
<td>2475</td>
</tr>
<tr>
<td>4</td>
<td>1925</td>
</tr>
<tr>
<td>Expression</td>
<td>$t$</td>
</tr>
<tr>
<td></td>
<td>$-550t + 4125$</td>
</tr>
</tbody>
</table>
4. A fish tank filled with 12 gallons of water is drained. The water drains at a rate of 1.5 gallons per minute.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td>Units</td>
<td>Volume</td>
</tr>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>10.5</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
</tr>
<tr>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Expression: \[-1.5t + 12\]

5. A submarine is traveling at a depth of −300 feet. It begins ascending at a rate of 28 feet per minute.

<table>
<thead>
<tr>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>Time</td>
</tr>
<tr>
<td>Units</td>
<td>Depth</td>
</tr>
<tr>
<td>0</td>
<td>−300</td>
</tr>
<tr>
<td>2</td>
<td>−244</td>
</tr>
<tr>
<td>4</td>
<td>−188</td>
</tr>
<tr>
<td>6</td>
<td>−132</td>
</tr>
<tr>
<td>8</td>
<td>−76</td>
</tr>
</tbody>
</table>

Expression: \[28t - 300\]
6. A free-diver is diving from the surface of the water at a rate of 15 feet per minute.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Independent Quantity</th>
<th>Dependent Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Time</td>
<td>Depth</td>
</tr>
<tr>
<td></td>
<td>minutes</td>
<td>feet</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>-15</td>
</tr>
<tr>
<td>2</td>
<td>-30</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>-45</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>-60</td>
<td>4</td>
</tr>
<tr>
<td>Expression</td>
<td>t</td>
<td>-15t</td>
</tr>
</tbody>
</table>

Identify the input value, the output value, the y-intercept, and the rate of change for each function.

7. A hot air balloon at 130 feet begins to ascend. It ascends at a rate of 160.5 feet per minute.

The function \( f(t) = 160.5t + 130 \) represents the height of the balloon as it ascends.

The input value is \( t \), time in minutes. The output value is \( f(t) \), height in feet.

The y-intercept is 130. The rate of change is 160.5.

8. A backyard pool contains 500 gallons of water. It is filled with additional water at a rate of 6 gallons per minute. The function \( f(t) = 6t + 500 \) represents the volume of water in the pool as it is filled.

The input value is \( t \), time in minutes. The output value is \( f(t) \), volume in gallons.

The y-intercept is 500. The rate of change is 6.

9. A submarine is diving from the surface of the water at a rate of 17 feet per minute. The function \( f(t) = -17t \) represents the depth of the submarine as it dives.

The input value is \( t \), time in minutes. The output value is \( f(t) \), depth in feet.

The y-intercept is 0. The rate of change is \(-17\).
10. A helicopter flying at 3505 feet begins its descent. It descends at a rate of 470 feet per minute. The function \( f(t) = -470t + 3505 \) represents the height of the helicopter as it descends.

   The input value is \( t \), time in minutes. The output value is \( f(t) \), height in feet.

   The \( y \)-intercept is 3505. The rate of change is \(-470\).

11. A bathtub contains 5 gallons of water. The faucet is turned on and water is added to the tub at a rate of 4.25 gallons per minute. The function \( f(t) = 4.25t + 5 \) represents the volume of water in the bathtub as it is filled.

   The input value is \( t \), time in minutes. The output value is \( f(t) \), volume in gallons.

   The \( y \)-intercept is 5. The rate of change is 4.25.

12. A free-diver is diving from the surface of the water at a rate of 8 feet per minute. The function \( f(t) = -8t \) represents the depth of the diver.

   The input value is \( t \), time in minutes. The output value is \( f(t) \), depth in feet.

   The \( y \)-intercept is 0. The rate of change is \(-8\).

Sketch the line for the dependent value to estimate each intersection point.

### Graphs

**Graph 1**
- \( f(x) = -40x + 1200 \) when \( f(x) = 720 \)

**Graph 2**
- \( f(x) = 6x + 15 \) when \( f(x) = 75 \)

**Answers**

- For Graph 1: \( f(x) = 720 \text{ at } x = 12 \)
- For Graph 2: \( f(x) = 75 \text{ at } x = 10 \)

Answers will vary.
15. \( f(x) = -2x + 5 \) when \( f(x) = -7 \)

16. \( f(x) = 4x - 7 \) when \( f(x) = 8 \)

Answers will vary.

17. \( f(x) = -200x + 2400 \) when \( f(x) = 450 \)

18. \( f(x) = 12x + 90 \) when \( f(x) = 420 \)

Answers will vary.

Answers will vary.

Answers will vary.

Answers will vary.
Name _____________________________  Date ___________

Substitute and solve for \( x \) to determine the exact value of each intersection point.

19. \( f(x) = -40x + 1200 \) when \( f(x) = 720 \)
   \[
   \begin{align*}
   f(x) &= -40x + 1200 \\
   720 &= -40x + 1200 \\
   -480 &= -40x \\
   12 &= x
   \end{align*}
   \]

20. \( f(x) = 6x + 15 \) when \( f(x) = 75 \)
   \[
   \begin{align*}
   f(x) &= 6x + 15 \\
   75 &= 6x + 15 \\
   60 &= 6x \\
   10 &= x
   \end{align*}
   \]

21. \( f(x) = -2x + 5 \) when \( f(x) = -7 \)
   \[
   \begin{align*}
   f(x) &= -2x + 5 \\
   -7 &= -2x + 5 \\
   -12 &= -2x \\
   6 &= x
   \end{align*}
   \]

22. \( f(x) = 4x - 7 \) when \( f(x) = 8 \)
   \[
   \begin{align*}
   f(x) &= 4x - 7 \\
   8 &= 4x - 7 \\
   15 &= 4x \\
   3.75 &= x
   \end{align*}
   \]

23. \( f(x) = -200x + 2400 \) when \( f(x) = 450 \)
   \[
   \begin{align*}
   f(x) &= -200x + 2400 \\
   450 &= -200x + 2400 \\
   -1950 &= -200x \\
   9.75 &= x
   \end{align*}
   \]

24. \( f(x) = 12x + 90 \) when \( f(x) = 420 \)
   \[
   \begin{align*}
   f(x) &= 12x + 90 \\
   420 &= 12x + 90 \\
   330 &= 12x \\
   27.5 &= x
   \end{align*}
   \]
Scouting for Prizes!
Modeling Linear Inequalities

Vocabulary
Define the term in your own words.

1. solve an inequality
   To solve an inequality means to determine the values of the variable that make the inequality true.

Problem Set
Carlos works at an electronics store selling computer equipment. He can earn a bonus if he sells $10,000 worth of computer equipment this month. So far this month, he has sold $4000 worth of computer equipment. He hopes to sell additional laptop computers for $800 each to reach his goal. The function $f(x) = 800x + 4000$ represents Carlos’s total sales as a function of the number of laptop computers he sells.
Use the graph to write an equation or inequality to determine the number of laptop computers Carlos would need to sell to earn each amount.

1. at least $10,000
   Carlos would need to sell at least 8 laptop computers.
   \[ x \geq 8 \]

2. less than $7000
   Carlos would need to sell fewer than 4 laptop computers.
   \[ x < 4 \]

3. less than $6000
   Carlos would need to sell fewer than 3 laptop computers.
   \[ x < 3 \]

4. at least $9000
   Carlos would need to sell at least 7 laptop computers.
   \[ x \geq 7 \]

5. more than $12,000
   Carlos would need to sell more than 10 laptop computers.
   \[ x > 10 \]

6. exactly $8000
   Carlos would need to sell exactly 5 laptop computers.
   \[ x = 5 \]

Elena works at the ticket booth of a local playhouse. On the opening night of the play, tickets are $10 each. The playhouse has already sold $500 worth of tickets during a presale. The function \( f(x) = 10x + 500 \) represents the total sales as a function of tickets sold on opening night.
Use the graph of the function to answer each question. Graph each solution on the number line.

7. How many tickets must Elena sell in order to make at least $1000?
   Elena must sell at least 50 tickets. \( x \geq 50 \)

8. How many tickets must Elena sell in order to make less than $800?
   Elena must sell fewer than 30 tickets. \( x < 30 \)

9. How many tickets must Elena sell in order to make at least $1200?
   Elena must sell at least 70 tickets. \( x \geq 70 \)

10. How many tickets must Elena sell in order to make exactly $1400?
    Elena must sell exactly 90 tickets. \( x = 90 \)

11. How many tickets must Elena sell in order to make less than $600?
    Elena must sell fewer than 10 tickets. \( x < 10 \)

12. How many tickets must Elena sell in order to make exactly $900?
    Elena must sell exactly 40 tickets. \( x = 40 \)
Leon plays on the varsity basketball team. So far this season he has scored a total of 52 points. He scores an average of 13 points per game. The function \( f(x) = 13x + 52 \) represents the total number of points Leon will score this season. Write and solve an inequality to answer each question.

13. How many more games must Leon play in order to score at least 117 points?

\[
\begin{align*}
117 & \leq 13x + 52 \\
65 & \leq 13x \\
5 & \leq x
\end{align*}
\]

Leon must play in 5 or more games to score at least 117 points.

14. How many more games must Leon play in order to score fewer than 182 points?

\[
\begin{align*}
182 & > 13x + 52 \\
130 & > 13x \\
10 & > x
\end{align*}
\]

Leon must play in fewer than 10 games to score fewer than 182 points.

15. How many more games must Leon play in order to score more than 143 points?

\[
\begin{align*}
143 & < 13x + 52 \\
91 & < 13x \\
7 & < x
\end{align*}
\]

Leon must play in more than 7 games to score more than 143 points.
LESSON 2.3 Skills Practice

16. How many more games must Leon play in order to score at least 100 points?
   \[ f(x) = 13x + 52 \]
   \[ 100 \leq 13x + 52 \]
   \[ 48 \leq 13x \]
   \[ 3.69 \leq x \]
   Leon must play in 4 or more games to score at least 100 points.

17. How many more games must Leon play in order to score fewer than 85 points?
   \[ f(x) = 13x + 52 \]
   \[ 85 > 13x + 52 \]
   \[ 33 > 13x \]
   \[ 2.54 > x \]
   Leon must play in 2 or fewer games to score fewer than 85 points.

18. How many more games must Leon play in order to score more than 200 points?
   \[ f(x) = 13x + 52 \]
   \[ 200 < 13x + 52 \]
   \[ 148 < 13x \]
   \[ 11.38 < x \]
   Leon must play in 12 or more games to score more than 200 points.
Draw an oval on the graph to represent the solution to each question. Write the corresponding inequality statement.

19. A hot air balloon at 4000 feet begins its descent. It descends at a rate of 200 feet per minute. The function \( f(x) = -200x + 4000 \) represents the height of the balloon as it descends. How many minutes have passed if the balloon is below 3000 feet?

More than 5 minutes have passed if the balloon is below 3000 feet. \( x > 5 \)

20. A bathtub filled with 55 gallons of water is drained. The water drains at a rate of 5 gallons per minute. The function \( f(x) = -5x + 55 \) represents the volume of water in the tub as it drains. How many minutes have passed if the tub still has more than 20 gallons of water remaining in it?

Less than 7 minutes have passed if the tub has more than 20 gallons of water remaining in it. \( x < 7 \)
21. Lea is walking to school at a rate of 250 feet per minute. Her school is 5000 feet from her home. The function \( f(x) = 250x \) represents the distance Lea walks. How many minutes have passed if Lea still has more than 2000 feet to walk?

Less than 12 minutes have passed if Lea still has more than 2000 feet to walk. \( x < 12 \)

22. Franco is riding his bike to school at a rate of 600 feet per minute. His school is 9000 feet from his home. The function \( f(x) = 600x \) represents the distance Franco rides. How many minutes have passed if Franco has less than 3000 feet left to ride?

More than 10 minutes have passed if Franco has less than 3000 feet to ride. \( x > 10 \)
23. A submarine is diving from the surface of the water at a rate of 20 feet per minute. The function \( f(x) = -20x \) represents the depth of the submarine as it dives. How many minutes have passed if the submarine is at least 160 feet below the surface?

At least 8 minutes have passed if the submarine is at least 160 feet below the surface.

\( x \geq 8 \)

24. A scuba diver is diving from the surface of the water at a rate of 14 feet per minute. The function \( f(x) = -14x \) represents the depth of the diver as he dives. How many minutes have passed if the diver is less than 42 feet below the surface?

Less than 3 minutes have passed if the diver is less than 42 feet below the surface.

\( x < 3 \)
We’re Shipping Out!
Solving and Graphing Compound Inequalities

Vocabulary
Match each definition to its corresponding term.
1. compound inequality
   a. a solution of a compound inequality in the form $a < x < b$, where $a$ and $b$ are any real numbers
   b. an inequality that is formed by the union, “or,” or the intersection, “and,” of two simple inequalities
2. solution of a compound inequality
   c. the part or parts of the solutions that satisfy both of the inequalities
   d. a solution of a compound inequality in the form $x < a$ or $x > b$, where $a$ and $b$ are any real numbers
3. conjunction
4. disjunction

Problem Set
Write each compound inequality in compact form.

1. All numbers less than or equal to 22 and greater than $-4$
   $22 \geq x > -4$
2. All numbers less than 55 and greater than 45
   $55 > x > 45$
3. All numbers greater than or equal to 0 and less than or equal to 6
   $0 \leq x \leq 6$
4. All numbers greater than 10 and less than 1000
   $10 < x < 1000$
5. All numbers less than or equal to 87 and greater than or equal to 83
   $87 \geq x \geq 83$
6. All numbers greater than $-1$ and less than or equal to 39
   $-1 < x \leq 39$
Write an inequality for each graph.

7. \(-8 < x \leq 11\)

8. \(2 \leq x \leq 6\)

9. \(7 < x < 25\)

10. \(-5 < x < 9\)

11. \(-14 \leq x \leq 5\)

12. \(-2 < x \leq 18\)

Graph each inequality.

13. \(45 < x < 75\)

14. \(-5 < x < 5\)

15. \(-13 \leq x \leq 5\)
Write a compound inequality for each situation.

19. The flowers in the garden are 6 inches or taller or shorter than 3 inches.
   \[ x \geq 6 \text{ or } x < 3 \]

20. People with a driver's license are at least 16 years old and no older than 85 years old.
   \[ 16 \leq x \leq 85 \]

21. Kyle's car gets more than 31 miles per gallon on the highway or 26 miles or less per gallon in the city.
   \[ x > 31 \text{ or } x \leq 26 \]

22. The number of houses that will be built in the new neighborhood must be at least 14 and no more than 28.
   \[ 14 \leq x \leq 28 \]

23. At the High and Low Store, they sell high-end items that sell for over $1000 and low-end items that sell for less than $10.
   \[ x > 1000 \text{ or } x < 10 \]

24. The heights of the twenty tallest buildings in New York City range from 229 meters to 381 meters.
   \[ 229 \leq x \leq 381 \]
Represent the solution to each part of the compound inequality on the number line. Then write the final solution that is represented by each graph.

25. \( x > 2 \) and \( x \leq 7 \)

\[ \begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\end{array} \]

\( 2 < x \leq 7 \)

26. \( x > 10 \) or \( x > 6 \)

\[ \begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
\hline
\end{array} \]

\( x > 6 \)

27. \( x \geq 5 \) or \( x < 3 \)

\[ \begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\end{array} \]

\( x \geq 5 \) or \( x < 3 \)

28. \( x > 4 \) and \( x < 3 \)

\[ \begin{array}{c}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
\end{array} \]

No solution

29. \( x \leq -1 \) or \( x > 0 \)

\[ \begin{array}{c}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\end{array} \]

\( x \leq -1 \) or \( x > 0 \)
30. $8 > x \geq -8$

$8 > x \geq -8$

31. $x \leq 9$ and $x \geq 2$

$2 \leq x \leq 9$

32. $x > -11$ or $x \leq -11$

All real numbers

Solve each compound inequality. Then graph and describe the solution.

33. $-3 < x + 7 \leq 17$

$-3 < x + 7 \leq 17$

$-3 - 7 < x + 7 - 7 \leq 17 - 7$

$-10 < x \leq 10$

Solution: $-10 < x \leq 10$
34. \[ 4 \leq 2x + 2 < 12 \]
\[ 4 \leq 2x + 2 < 12 \]
\[ 4 - 2 \leq 2x + 2 - 2 < 12 - 2 \]
\[ 2 \leq 2x < 10 \]
\[ \frac{2}{2} \leq \frac{2x}{2} < \frac{10}{2} \]
\[ 1 \leq x < 5 \]

Solution: \[ 1 \leq x < 5 \]

35. \[ x + 5 > 14 \text{ or } 3x < 9 \]
\[ x + 5 > 14 \]
\[ x + 5 - 5 > 14 - 5 \]
\[ x > 9 \]
\[ 3x < 9 \]
\[ \frac{3x}{3} < \frac{9}{3} \]
\[ x < 3 \]

Solution: \[ x > 9 \text{ or } x < 3 \]

36. \[ -5x + 1 \geq 16 \text{ or } x - 6 \leq -8 \]
\[ -5x + 1 \geq 16 \]
\[ -5x + 1 - 1 \geq 16 - 1 \]
\[ -5x \geq 15 \]
\[ \frac{-5x}{-5} \leq \frac{15}{-5} \]
\[ x \leq -3 \]
\[ x - 6 \leq -8 \]
\[ x - 6 + 6 \leq -8 + 6 \]
\[ x \leq -2 \]

Solution: \[ x \leq -2 \]
LESSON 2.4 Skills Practice

Name _____________________________ Date _____________

37. \( \frac{7}{8}x < 42 \)

\[
28 \leq \frac{7}{8}x < 42
\]

\[
\frac{8}{7}(28) \leq \frac{8}{7}\left(\frac{7}{8}x\right) < \frac{8}{7}(42)
\]

\(32 \leq x < 48\)

Solution: \(32 \leq x < 48\)

38. \(-2x + 5 \leq 9\) or \(-x - 13 > -31\)

\(-2x + 5 \leq 9\) or \(-x - 13 > -31\)

\[-2x + 5 \leq 9\]

\[-2x \leq 4\]

\[-x \leq -2\]

\[x \geq -2\]

\[-x - 13 > -31\]

\[-x > -18\]

\[-x < -18\]

\[x < 18\]

Solution: All real numbers
Play Ball!
Absolute Value Equations and Inequalities

Vocabulary
Define each term in your own words.

1. opposites
   Opposites are two numbers that are equal distance, but are in different directions, from zero on the number line.

2. absolute value
   The absolute value of a number is its distance from zero on the number line.

Give an example of each term.

3. linear absolute value equation
   Answers will vary.
   \[|x + 2| = 4\]

4. linear absolute value inequality
   Answers will vary.
   \[|x - 3| > 8\]

Match each equivalent compound inequality to its corresponding absolute value inequality.

5. \[|ax + b| < c\]
   a. \[-c < ax + b < c\]
   b. \[ax + b < -c\] or \[ax + b > c\]
   c. \[-c \leq ax + b \leq c\]
   d. \[ax + b \leq -c\] or \[ax + b \geq c\]

Problem Set
Evaluate each absolute value.

1. \[|3| = 3\]
2. \[|-3| = 3\]
3. \[\left|\frac{1}{4}\right| = \frac{1}{4}\]
4. \[\left|\frac{-1}{4}\right| = \frac{1}{4}\]
5. \[|3.7| = 3.7\]
6. \[|-3.7| = 3.7\]
Determine the number of solutions for each equation. Then calculate the solution.

7. \( x = -9 \)
   There is only one solution.
   \( x = -9 \)

8. \(|x| = -6\)
   There are no solutions.

9. \(|x| = 4\)
   There are two solutions.
   \( x = 4 \) or \( x = -4 \)

10. \(|-x| = -8\)
    There are no solutions.

11. \(|x| = 0\)
    There is only one solution.
    \( x = 0 \)

12. \(|-x| = 15\)
    There are two solutions.
    \( x = 15 \) or \( x = -15 \)

Solve each linear absolute value equation.

13. \(|x + 9| = 2\)
    \((x + 9) = 2\)
    \( x + 9 - 9 = 2 - 9 \)
    \( x = -7 \)

14. \(|x + 4| = 10\)
    \((x + 4) = 10\)
    \( x + 4 - 4 = 10 - 4 \)
    \( x = 6 \)

15. \(|x - 12| = 5\)
    \((x - 12) = 5\)
    \( x - 12 + 12 = 5 + 12 \)
    \( x = 17 \)

16. \(|2x - 6| = 18\)
    \((2x - 6) = 18\)
    \( 2x - 6 + 6 = 18 + 6 \)
    \( 2x = 24 \)
    \( x = 12 \)
Name _____________________________________ Date ____________

17. \(|3x + 1| = -9\)

There are no solutions.

18. \(|5x + 1| = 14\)

\[
\begin{align*}
(5x + 1) &= 14 \\
5x + 1 - 1 &= 14 - 1 \\
x &= 13 \\
&= \frac{13}{5}
\end{align*}
\]

\[
\begin{align*}
-(5x + 1) &= 14 \\
5x + 1 &= -14 \\
x &= -3
\end{align*}
\]

Solve each linear absolute value equation.

19. \(|x - 8| = 25\)

\[
\begin{align*}
|x - 8| &= 25 \\
x - 8 &= 25 \\
|x - 8 + 8 &= 25 + 8 \\
|x| &= 33 \\
x &= 33 \\
&= -33
\end{align*}
\]

20. \(|x + 3| - 7 = 40\)

\[
\begin{align*}
|x + 3| - 7 &= 40 \\
|x + 3| - 7 + 7 &= 40 + 7 \\
|x + 3| &= 47 \\
(x + 3) &= 47 \\
x + 3 - 3 &= 47 - 3 \\
x &= 44 \\
&= -50
\end{align*}
\]

21. \(2|x - 6| = 48\)

\[
\begin{align*}
2|x - 6| &= 48 \\
\frac{2|x - 6|}{2} &= \frac{48}{2} \\
|x - 6| &= 24 \\
-(x - 6) &= 24 \\
(x - 6) &= 24 \\
x - 6 + 6 &= 24 + 6 \\
x &= 30 \\
x &= -18
\end{align*}
\]
22. \(3|x + 8| = 36\)

\[
3|x + 8| = 36 \\
\frac{3|x + 8|}{3} = \frac{36}{3} \\
|x + 8| = 12 \\
(x + 8) = 12 \\
x + 8 - 8 = 12 - 8 \\
x = 4
\]

\[-(x + 8) = 12 \\
x + 8 = -12 \\
x + 8 - 8 = -12 - 8 \\
x = -20
\]

23. \(5|x| + 4 = 79\)

\[
5|x| + 4 = 79 \\
5|x| + 4 - 4 = 79 - 4 \\
5|x| = 75 \\
\frac{5|x|}{5} = \frac{75}{5} \\
|x| = 15 \\
-(x) = 15 \\
x = -15
\]

24. \(2|x| - 5 = 11\)

\[
2|x| - 5 = 11 \\
2|x| - 5 + 5 = 11 + 5 \\
2|x| = 16 \\
\frac{2|x|}{2} = \frac{16}{2} \\
|x| = 8 \\
-(x) = 8 \\
x = -8
\]

Solve each linear absolute value inequality. Graph the solution on the number line.

25. \(|x + 5| < 2\)

\[
(x + 5) < 2 \\
-(x + 5) < 2 \\
x + 5 - 5 < 2 - 5 \\
x < -3 \\
x + 5 - 5 < -2 - 5 \\
x + 5 > -2 \\
x > -7
\]
Name _____________________________ Date _____________

26. $|x - 3| \leq 6$

$(x - 3) \leq 6$
$x - 3 + 3 \leq 6 + 3$
$x \leq 9$

$-(x - 3) \leq 6$
$x - 3 \geq -6$
$x - 3 + 3 \geq -6 + 3$
$x \geq -3$

27. $2|x - 1| < 14$

$2|x - 1| < 14$
$\frac{2|x - 1|}{2} < \frac{14}{2}$
$|x - 1| < 7$
$(x - 1) < 7$
$x - 1 + 1 < 7 + 1$
$x < 8$

$-(x - 1) < 7$
$x - 1 > -7$
$x - 1 + 1 > -7 + 1$
$x > -6$

28. $3|x + 4| \geq 9$

$3|x + 4| \geq 9$
$\frac{3|x + 4|}{3} \geq \frac{9}{3}$
$|x + 4| \geq 3$
$(x + 4) \geq 3$
$x + 4 - 4 \geq 3 - 4$
$x \geq -1$

$-(x + 4) \geq 3$
$x + 4 \leq -3$
$x + 4 - 4 \leq -3 - 4$
$x \leq -7$
29. $2|x - 1| - 8 \leq 10$

\[ 2|x - 1| - 8 \leq 10 \]
\[ 2|x - 1| - 8 + 8 \leq 10 + 8 \]
\[ 2|x - 1| \leq 18 \]
\[ \frac{2|x - 1|}{2} \leq \frac{18}{2} \]
\[ |x - 1| \leq 9 \]
\[ (x - 1) \leq 9 \]
\[ x - 1 + 1 \leq 9 + 1 \]
\[ x \leq 10 \]
\[ -(x - 1) \leq 9 \]
\[ x - 1 \geq -9 \]
\[ x - 1 + 1 \geq -9 + 1 \]
\[ x \geq -8 \]

30. $3|x + 2| + 5 \geq 23$

\[ 3|x + 2| + 5 \geq 23 \]
\[ 3|x + 2| + 5 - 5 \geq 23 - 5 \]
\[ 3|x + 2| \geq 18 \]
\[ \frac{3|x + 2|}{3} \geq \frac{18}{3} \]
\[ |x + 2| \geq 6 \]
\[ -(x + 2) \geq 6 \]
\[ (x + 2) \geq 6 \]
\[ x + 2 - 2 \geq 6 - 2 \]
\[ x \geq 4 \]
\[ x + 2 - 2 \leq -6 - 2 \]
\[ x + 2 \leq -6 \]
\[ x \leq -8 \]
Graph the function that represents each problem situation. Draw an oval on the graph to represent the answer.

31. A jewelry company is making 16-inch bead necklaces. The specifications allow for a difference of 0.5 inch. The function $f(x) = |x - 16|$ represents the difference between the necklaces manufactured and the specifications. Graph the function. What necklace lengths meet the specifications?
   The necklaces can be between 15.5 and 16.5 inches long to meet the specifications.

32. Julian is cutting lengths of rope for a class project. Each rope length should be 10 inches long. The specifications allow for a difference of 1 inch. The function $f(x) = |x - 10|$ represents the difference between the rope lengths cut and the specifications. Graph the function. What rope lengths meet the specifications?
   The rope lengths can be between 9 and 11 inches long to meet the specifications.

33. A snack company is filling bags with pita chips sold by weight. Each bag should contain 8 ounces of chips. The specifications allow for a difference of 0.25 ounce. The function $f(x) = |x - 8|$ represents the difference between the weight of a bag of chips and the specifications. Graph the function. What weights meet the specifications?
   Each bag of chips can weigh between 7.75 ounces and 8.25 ounces.
34. A cereal company is filling boxes with cereal sold by weight. Each box should contain 32 ounces of cereal. The specifications allow for a difference of 0.5 ounce. The function $f(x) = |x - 32|$ represents the difference between the weight of a box of cereal and the specifications. Graph the function. What weights do not meet the specifications?

A box of cereal that weighs less than 31.5 ounces or more than 32.5 ounces does not meet the specifications.

35. Guests at the school harvest festival are asked to guess how many peanuts are in a jar. The jar contains 260 peanuts. All guests within 10 peanuts of the correct answer win a prize. The function $f(x) = |x - 260|$ represents the difference between a guess and the actual number of peanuts in the jar. Graph the function. What possible guesses will not win a prize?

A guess that is more than 270 or less than 250 will not win a prize.

36. The rules of an art contest state that sculptures submitted should be 3 feet high but allow for a difference of 6 inches. The function $f(x) = |x - 3|$ represents the difference between a sculpture that is submitted and the specifications. Graph the function. What heights do not meet the specifications?

A sculpture that is shorter than 2.5 feet or taller than 3.5 feet does not meet the specifications.
Choose Wisely!
Understanding Non-Linear Graphs and Inequalities

Problem Set
Choose the function that represents each problem situation.

1. Tonya is walking to school at a rate of 3 miles per hour.
   A \( f(x) = 3x^2 \)  
   B \( f(x) = 3x \)  
   C \( f(x) = 3^x \)

2. Guests at a craft fair are asked to guess how many beads are in a jar. The jar contains 220 beads.
   All guests within 10 beads of the correct answer win a prize.
   A \( f(x) = |x - 220| \)  
   B \( f(x) = 220 - x \)  
   C \( f(x) = 220^x \)
   A \( f(x) = |x - 220| \)

3. Mario buys a car for $25,000. Each year the car loses \( \frac{1}{6} \) of its value.
   A \( f(x) = 25,000 - \frac{1}{6}x \)  
   B \( f(x) = \frac{1}{6}x^2 + 25,000 \)  
   C \( f(x) = 25,000\left(\frac{5}{6}\right)^x \)

4. A bathtub filled with 50 gallons of water is drained. The water drains at a rate of 5 gallons per minute.
   A \( f(x) = 50 - 5x \)  
   B \( f(x) = 5x^2 - 50 \)  
   C \( f(x) = 50 - 5^x \)
   A \( f(x) = 50 - 5x \)

5. Rodell throws a football straight up with a speed of 25 feet per second. The acceleration of the ball due to gravity is 32 feet per second.
   A \( f(x) = -32x + 25 \)  
   B \( f(x) = -32x^2 + 25x \)  
   C \( f(x) = |32x - 25| \)
   B \( f(x) = -32x^2 + 25x \)

6. A pasta company is filling boxes with pasta sold by weight. Each box should contain 16 ounces of pasta. The specifications allow for a difference of 0.5 ounce.
   A \( f(x) = 16x - 0.5 \)  
   B \( f(x) = 16x^2 - 0.5x \)  
   C \( f(x) = |x - 16| \)
   C \( f(x) = |x - 16| \)
Graph the function that represents each problem situation. Use the graph to answer the question.

7. A fish tank filled with 20 gallons of water is drained. The water drains at a rate of 4 gallons per minute. The function \( f(x) = 20 - 4x \) represents the volume of water in the fish tank as it drains. Graph the function. How many minutes does it take for half of the water to drain from the tank?

After 2.5 minutes, half of the water in the tank (10 gallons) will be drained.

8. A pasta company is filling boxes with pasta sold by weight. Each box should contain 32 ounces of pasta. The specifications allow for a difference of 1.5 ounces. The function \( f(x) = |x - 32| \) represents the difference between the weight of a box of pasta and the specifications. Graph the function. What weights meet the specifications?

Weights between 30.5 and 33.5 ounces meet the specifications.
9. Ronna buys a car for $20,000. Each year the car loses \( \frac{1}{4} \) of its value. The function \( f(x) = 20,000 \left( 3^x \right) \frac{3}{4} \) represents the value of the car over time. Graph the function. Ronna wants to eventually sell the car and make at least $10,000 in the sale. Estimate the number of years Ronna can own the car before she must resell and still make at least $10,000.

Ronna can own the car for almost 2.5 years before reselling and will still make at least $10,000.

10. Serena is driving to her aunt’s house at a rate of 55 miles per hour. The function \( f(x) = 55x \) represents the distance Serena travels over time. Graph the function. Estimate how long it will take Serena to get to her aunt’s house which is 192 miles away.

It will take Serena about 3.5 hours to reach her aunt’s house.
11. Hector throws a softball straight up with a speed of 50 feet per second. The acceleration of the ball due to gravity is 32 feet per second. The function \( f(x) = -32x^2 + 50x \) represents the height of the softball as it travels up in the air and back to the ground. Graph the function. Estimate the length of time the softball is in the air.

The softball is in the air for about 1.5 seconds.

12. Guests at a craft fair are asked to guess how many beads are in a jar. The jar contains 180 beads. All guests within 20 beads of the correct answer win a prize. The function \( f(x) = |x - 180| \) represents the difference between a guess and the actual number of beads in the jar. Graph the function. What possible guesses will win a prize?

Guesses that are between 160 beads and 200 beads will win a prize.