The Playoffs
Graphing Inequalities

1. Jeremy is working at two jobs to save money for his college education. He makes $8 per hour working for his uncle at Pizza Pie busing tables and $10 per hour tutoring peers after school in math. His goal is to make $160 per week.
   a. If Jeremy works 8 hours at Pizza Pie and tutors 11 hours during the week, does he reach his goal?
      
      \[8(8) + 10(11) = 174\]
      
      Yes. Jeremy exceeds his goal of making $160 for the week.

   b. Write an expression to represent the total amount of money Jeremy makes in a week from working both jobs. Let \(x\) represent the number of hours he works at Pizza Pie and \(y\) represent the number of hours he tutors.
      
      \[8x + 10y\]

   c. After researching the costs of colleges, Jeremy decides he needs to make more than $160 each week. Write an inequality in two variables to represent the amount of money Jeremy needs to make.
      
      \[8x + 10y > 160\]

   d. Graph the inequality from part (c).
e. Is the point \((0, 0)\) in the shaded region of the graph? Explain why or why not.

No. The point \((0, 0)\) is not in the shaded region because it is not a solution to the inequality.

\[8(0) + 10(0) \geq 160\]

0 is not greater than 160.

f. According to the graph, if Jeremy works 5 hours at Pizza Pie and tutors for 10 hours, will he make more than $160? Explain.

No. Jeremy will not make more than $160 if he works 5 hours at Pizza Pie and tutors for 10 hours. The point \((5, 10)\) on the graph is not in the shaded solution region.

g. Due to days off from school, Jeremy will only be tutoring for 6 hours this week. Use the graph to determine the least amount of full hours he must work at Pizza Pie to still reach his goal. Then show that your result satisfies the inequality.

Jeremy must work at least 13 hours at Pizza Pie in order to make more than 160 dollars for the week.

\[8(13) + 10(6) > 160\]

104 + 60 > 160

164 > 160
Lesson 7.2 Assignment

Working the System
Systems of Linear Inequalities

1. Samuel is remodeling his basement. One part of the planning involves the flooring. He knows that he would like both carpet and hardwood, but isn’t sure how much of each he will use. The most amount of flooring area he can cover is 2000 square feet. The carpet is $4.50 per square foot and the hardwood is $8.25 per square foot. Both prices include labor costs. Samuel has budgeted $10,000 for the flooring.

   a. Write a system of inequalities to represent the maximum amount of flooring needed and the maximum amount of money Samuel wants to spend.

\[
\begin{align*}
   x + y &\leq 2000 \\
   4.50x + 8.25y &\leq 10,000
\end{align*}
\]

   b. One idea Samuel has is to make two rooms; one having 400 square feet of carpeting and the other having 1200 square feet of hardwood. Determine whether this amount of carpeting and hardwood are solutions to the system of inequalities. Explain your reasoning in terms of the problem situation.

\[
\begin{align*}
   x + y &\leq 2000 \\
   400 + 1200 &\leq 2000 \\
   1600 &\leq 2000 \\
   4.50(400) + 8.25(1200) &\leq 10,000 \\
   1800 + 9900 &\leq 10,000 \\
   11,700 &\neq 10,000
\end{align*}
\]

No. This is not a solution to this system of inequalities because this amount of carpet and hardwood only results in a true statement for one of the inequalities. This means that although it will not exceed the total amount of square footage available to finish in the basement, the amount of each puts the total cost for flooring over the maximum budget of $10,000.
c. Graph this system of inequalities.

![Graph of system of inequalities]

d. Determine the intersection point of the two lines. Is this a solution to this system of inequalities in terms of the problem situation?

The intersection point of this system of inequalities is \(1733\frac{1}{3}, 266\frac{2}{3}\). Although it does make sense that there can be \(1733\frac{1}{3}\) square feet of carpet and \(266\frac{2}{3}\) square feet of hardwood, Samuel will most likely have to buy the flooring as whole number values of square feet.

e. Identify two different solutions to the system of inequalities. Explain what the solutions represent in terms of the problem situation.

Answers will vary.

(1500, 250)
This solution means that Samuel can put down 1500 square feet of carpet and 250 square feet of hardwood and not have too much flooring while not going over the budget.

(0, 1000)
This solution means that Samuel can put down no carpeting and 1000 square feet of hardwood and not have too much flooring while not going over the budget.

f. Determine one combination of amounts of carpet and hardwood that is not a solution for the system of inequalities. Explain your reasoning.

Answers will vary.

The point (500, 1000) does not represent a solution.

\[\begin{align*}
  x + y &\leq 2000 \\
  500 + 1000 &\leq 2000 \\
  1500 &\leq 2000
\end{align*}\]

Although Samuel would not have too much flooring, he would go over budget if he bought 500 square feet of carpet and 1000 square feet of hardwood.
Our Biggest Sale of the Season!
Systems with More Than Two Linear Inequalities

1. The Brunstown Ballet Company needs to rent a venue for their Holiday Production of the Nutcracker. There are a number of arenas they are considering. The arenas have seating capacities that range from 800 to 1876 seats. The management of the Ballet Company knows the ticket sales may not be good this year but their goal is to sell between 65% and 90% of the available seats. Whichever arena they choose, one hundred seats must be set aside for the Ballet Company’s donors.

   a. Write a system of inequalities that represents the problem situation. Define your variables.

   Let $a$ represent the number of available seats and $s$ represent the number of seats sold.

   \[
   \begin{align*}
   a &\geq 700 \\
   a &\leq 1776 \\
   s &\leq 0.90a \\
   s &\geq 0.65a
   \end{align*}
   \]

   b. Graph each inequality on the grid shown. Include labels and units of measure for each axis.
c. One of the arenas they are considering has 1200 available seats. Determine the minimum and maximum number of seats they would need to sell in order for management to reach their goal.

\[ s \leq 0.65(1200) \]
\[ s \geq 780 \]

The minimum number of seats they would need to sell is 780.

\[ s \leq 0.90(1200) \]
\[ s \leq 1080 \]

The maximum number of seats they would need to sell is 1080.

d. If the company sold 900 seats, what is the range of seating capacities for the arenas they may have rented?

\[ 900 \leq 0.90a \]
\[ 900 \leq 0.90a \]
\[ 1000 \leq a \]

If they sold 900 tickets, then the minimum number of seats in the arena was 1000.

\[ 900 \geq 0.65a \]
\[ 900 \leq 0.65a \]
\[ 1384.6 \leq a \]

If they sold 900 tickets, then the maximum number of seats in the arena was 1384.

e. If they rented an arena that had a 1300-seat capacity and sold 800 tickets, would management reach their goal? Explain your reasoning.

No. Management would not reach their goal. The point (1300, 800) does not fall in the solution region, so they would not have sold at least 65% of the seats.
Take It to the Max . . . or Min
Linear Programming

The Smartway Rental Car Company has $180,000 to invest in the purchase of at most 16 cars of two different types, compact and full-size.

<table>
<thead>
<tr>
<th></th>
<th>Purchase Price</th>
<th>Rental Fee</th>
<th>Maintenance Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compact Car</td>
<td>$9000</td>
<td>$30</td>
<td>$8</td>
</tr>
<tr>
<td>Full-Size Car</td>
<td>$15,000</td>
<td>$48</td>
<td>$10</td>
</tr>
</tbody>
</table>

1. Due to demand, Smartway needs to purchase at least 5 compact cars.
   a. Identify the constraints as a system of linear inequalities. Define your variables.
      Let $c$ represent the number of compact cars purchased.
      Let $f$ represent the number of full-size cars purchased.
      $c \geq 5$
      $f \geq 0$
      $9000c + 15,000f \leq 180,000$
      $c + f \leq 16$

   b. Graph the solution set for the system of linear inequalities. Label all points of the intersection of the boundary lines.
c. Smartway Rental Car’s income comes from renting out their cars. How many of each type of car should they purchase if they want to maximize their income? What is the maximum income?

\[ I(c, f) = 30c + 48f \]

\[ I = (5, 0) = 30(5) + 48(0) = 150 \]
\[ I = (5, 9) = 30(5) + 48(9) = 582 \]
\[ I = (10, 6) = 30(10) + 48(6) = 588 \]
\[ I = (16, 0) = 30(16) + 48(0) = 480 \]

In order to maximize the income that the additional cars will bring in, they should purchase 10 compact cars and 6 full-size cars. The maximum daily income is $588.

d. In order to keep up with their competitors, Smartway must purchase at least 3 full-size cars and at least 5 compact cars.

Identify the constraints as a system of linear inequalities. Define your variables.

Let \( c \) represent the number of compact cars purchased.

Let \( f \) represent the number of full-size cars purchased.

\[ c \geq 5 \]
\[ f \geq 3 \]
\[ 9000c + 15,000f \leq 180,000 \]
\[ c + f \leq 16 \]

e. Graph the solution set for this system of linear inequalities.
f. Smartway Rental is still unable to keep up with their competitors so they are going to try and cut their maintenance fees to save money. How many of each type of car should they purchase to minimize their maintenance fees?

\[ M(c, f) = 8c + 10f \]

\[ M(5, 3) = 8(5) + 10(3) = 70 \]

\[ M(5, 9) = 8(5) + 10(9) = 130 \]

\[ M(10, 6) = 8(10) + 10(6) = 140 \]

\[ M(13, 3) = 8(13) + 10(3) = 134 \]

In order to minimize their maintenance fees they should purchase the minimum number of cars, 5 compact and 3 full-size. They will then pay $70 a day in maintenance fees.